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Effect of Supersubdivision of Graphs on the Dominating Set and the Chromatic Number of Graphs

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Abstract: In this paper, we have shown that the graph obtained by the supersubdivision of all edges of a complete bipartite graphs, path graphs and cycle graphs by a complete bipartite graphs have uniformity in the minimal dominating set and their chromatic number is always two.

Keywords: Supersubdivision, Dominating set, Minimal dominating set, Chromatic number, Graceful labeling.

1. Introduction: J.A. Gallian[1] has given a dynamic survey of labeling of graphs. The gracefulness of arbitrary supersubdivision of graphs and one vertex union of non-isomorphic complete bipartite graphs have been introduced by G. Sethuraman and P. Selvaraju [2,3]. They have proved that one vertex union of non-isomorphic complete bipartite graphs K_{2,m_i}, $1 \leq i \leq n$ at the apex vertex is graceful.

Definition 1.1 Let G be a graph with p vertices and q edges. A graph H is said to be a *supersubdivision* of G if H is obtained by replacing each and every edge of G by a complete bipartite graph $K_{2,m}$ for any m.

Definition 1.2 A *dominating set* for a graph $G = (V, E)$ is a subset D of V such that each and every vertex not in D is joined to at least one member of D by some edge. The *dominating number* is the number of vertices in the smallest dominating set of G.

Definition 1.3 A *minimal dominating set* is a subset D_1 of G containing minimum number of vertices such that each and every vertex of G must be adjacent to a vertex in D_1. The cardinality is the number of vertices in the minimal dominating set D_1 and is denoted by $\gamma(G)$.

Definition 1.4 The *chromatic number* β(G) of a graph G is the minimum number of colours required to colour the vertices of G so that no two adjacent vertices have the same colour.

Definition 1.5 A graph $G = (V(G), E(G))$ with p vertices and q edges is said to admit *graceful labeling* if $f : V(G) \rightarrow \{0,1,2,\ldots,q\}$ such that distinct vertices receive distinct numbers and $\{|f(u) - f(v)| / uv \in E(G)\} = \{1,2,3,\ldots,q\}$.

2. Main Results

The results for different cases are as follows:

Result 2.1 Supersubdivision of all the edges of $K_{2,n}$ by $K_{2,i}$, $1 \leq i \leq n$ is considered. The resultant graph so obtained has the value $(n+2)$ as its cardinality of the minimal dominating set and its chromatic number is 2. This is illustrated as follows:

Illustration 2.2 Consider a complete bipartite graph $K_{2,3}$ and the supersubdivision of each and every edge of $K_{2,3}$ by $K_{2,2}$. The minimal dominating set of the resulting graph G is 5 and its chromatic number is 2.

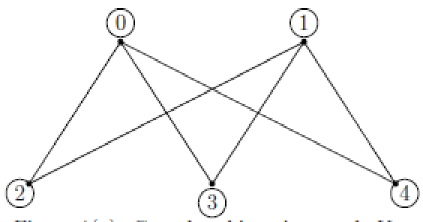

Figure 1(a). Complete bipartite graph $K_{2,3}$

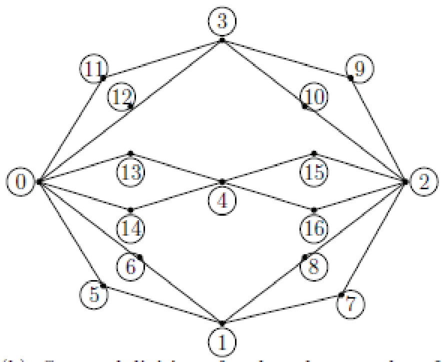

Figure 1(b). Supersubdivision of each and every edge of $K_{2,3}$ by $K_{2,2}$

Result 2.3 Consider the supersubdivision of all the edges of S_n by $K_{2,i}$, for any i and the resulting graph has the value $(n+1)$ as its cardinality of the minimal dominating set and the chromatic number is 2 again. This is illustrated as follows:

Illustration 2.4 Consider a star graph S_4 $(or\ K_{1,4})$ and the supersubdivision of all edges of $K_{1,4}$ namely OA, OB, OC, OD by $K_{2,2}, K_{2,3}, K_{2,4}, K_{2,5}$ respectively. The minimal dominating set of the resulting

graph G is 5 and its chromatic number is 2. This graph is also graceful which is shown in the Figure-2(b).

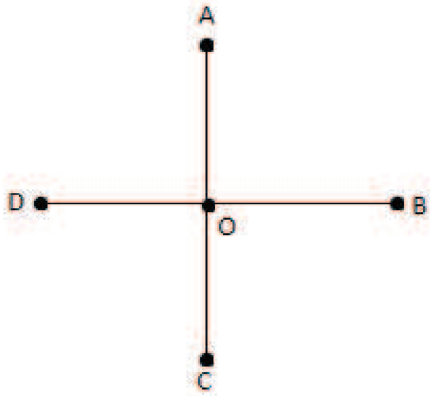

Figure 2(a) Star graph $K_{1,4}$

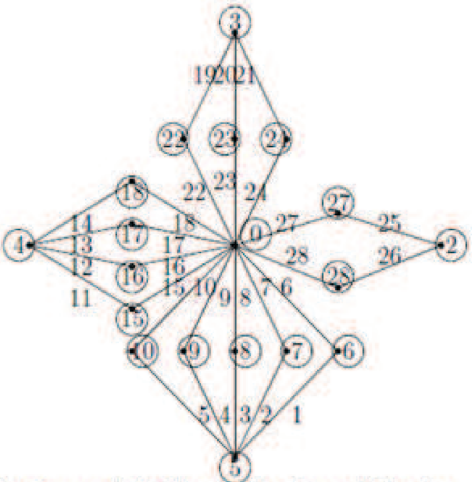

Figure 2(b) Supersubdivision of the edges of $K_{1,4}$ by $K_{2,2}$, $K_{2,3}$, $K_{2,4}$, $K_{2,5}$

Result 2.5 Consider the supersubdivision of all the edges of P_n arbitrarily by $K_{2,i}$, for any i and the resulting graph has the value n as its cardinality of the minimal dominating set and the chromatic number is 2 again. This is illustrated as follows.

Illustration 2.6 Consider a path graph P_4 and the supersubdivision of all edges of P_4 by $K_{2,2}, K_{2,3}, K_{2,5}$ respectively. The minimal dominating set of the resulting graph is 4 and its chromatic number is 2. The gracefulness of this graph is shown in Figure-3(b).

Figure 3(a). Path graph P_4

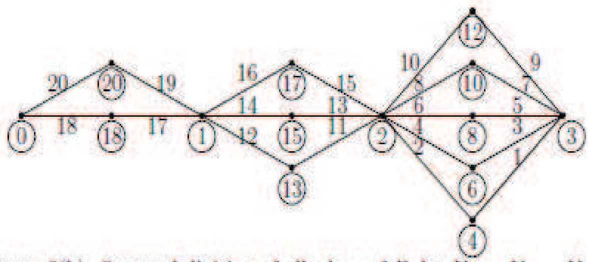

Figure 3(b). Supersubdivision of all edges of P_4 by $K_{2,2}$, $K_{2,3}$, $K_{2,5}$

Result 2.7 Consider the supersubdivision of all edges of C_n by $K_{2,i}$ for $1 \leq i \leq n$ and the resulting graph has the value n as its cardinality of the minimal dominating set and the chromatic number is 2. This is illustrated as follows:

Illustration 2.8 Consider a cycle graph C_3. We consider the supersubdivision of all edges of C_3 by $K_{2,2}$. We found that the minimal dominating set of the resulting graph G is 3 and its chromatic number is 2. This graph is also graceful as shown in Figure-4(b).

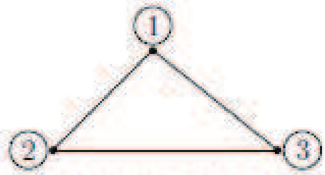

Figure 4(a). Cycle graph C_3

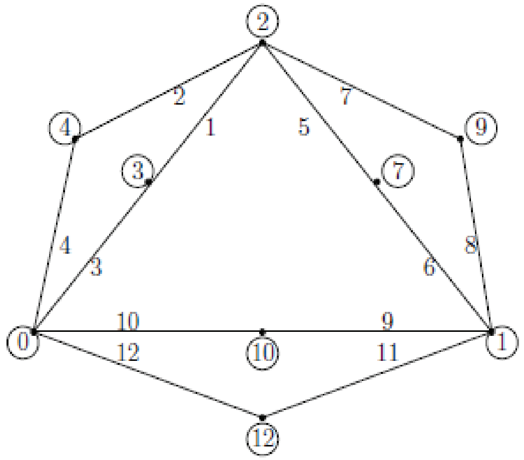

Figure 4(b). Supersubdivision of the edges of C_3 by $K_{2,2}$

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The results are tabulated as follows:

Graph	Supersubdivision of all edges by $K_{2,i}$,$1 \leq i \leq n$	$\Gamma(G)$	$\beta(G)$
$K_{2,n}$	$K_{2,i}$	$n+2$	2
S_n	$K_{2,i}$	$n+1$	2
P_n	$K_{2,i}$	n	2
C_n	$K_{2,i}$	n	2

$\boldsymbol{Table-1}$

where $\gamma(G)$ stands for cardinality of minimal dominating set and $\beta(G)$ stands for chromatic number of the resulting graph G.

5. CONCLUSION

The complete bipartite graphs, paths as well as cycles may by extended by supersubdivision of all the edges by a complete bipartite graph whose cardinality of the minimal dominating set and the chromatic number is shown in Table-1 in a compact form.

6. REFERENCES

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