# FRACTAL DIMENSION: A CASE STUDY ON BUTTERFLY WING

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**Abstract:** The geometrical complexity of butterfly wing pattern from different mimicry cases were analyzed in terms of their fractal dimension (FD), using box counting method programmed in Matlab (2012b). Fractals in simplistic term are geometrical figures which repeat themselves at progressively smaller scale and exhibit progressively more complex structure when observed at larger scale. Twenty different cases of mimics with models were put forward for the hypothesis for the significant difference in their mean FD values. The statistical data of t-test (Unpaired t-test, $p < 0.05$) and ANOVA (One way ANOVA, $p < 0.05$) along with data of effective size r shows the significant difference in mean FD value between mimics and models. Possibly accepted hypothesis stated that the FD can be used as an effective tool for the identification and to distinguish the butterflies from different taxa which show the mimicry and looks similar at the first site can be differentiated on the bases of fractality. Thus data presents the application of fractal dimension as an important aspect to identify the mimic species and to study the geometrical pattern of butterfly wing.

**Keywords:** Fractality, Fractal Diemsion, Matlab, Mimicry.

**Introduction:** Visual mimicry is a textbook case of natural selection because it is both intuitively understandable and has repeatedly evolved in a variety of organisms: it is the ultimate example of parallel evolution [12]. In many mimetic groups, particularly butterflies, a huge variety of color patterns has arisen, even in closely related species. Related butterfly species may possess quite different wing patterns particularly when only certain species, sexes or forms within group are involved in mimicry complex [9]. Examples of co-evolutionary pathways and taxonomic distribution of lepidopteron are common in literatures [10]. The genetic studies provide the evolutionary pattern of butterfly species [7]. Authentically there is no technique, which supports the identification of butterfly species which forms the mimicry ring.

With reference to the study of the subjective characters of members in mimicry ring, an analysis of the geometrical pattern of wing was carried out using the concept of modern mathematics on the basis of measurement of fractal dimension [1].

Withstanding this review, the present study was attempted to initiate the practice of implantation of mathematical tool on biological structure. Total twenty cases of butterfly are found to follow the subtle pattern of mimicry viz. Polymorphic or sex limited mimicry. The study was proposed to specify the Mimic and Model based on the obtained fractal dimension values of wing pattern. As explained in the next section, the fractal dimension contains geometrical information from all possible scales [4]. Thus this recent technique may provide an adequate tool for an identification and insulation of mimic species.

**Fractal dimension:** The term 'fractal' is coined by Mandelbrot (1977) means self similar pattern. A characteristic of fractal geometry is that the length of an object depends on the resolution or the scale at which the object is measured [4]. Many objects in nature show self-similarity. This fractality of an object can be measured in terms of their fractal dimension by using, well known Box-counting method [1].

For example, in fractal dimension calculating by Box-Counting dimension method, the space is divided into boxes with size of ε, and the boxes (N) can be counted as the fractality has effect on them. The fractal dimension can be calculated via relation (1). The result

$$D = \varepsilon \xrightarrow{lim} 0 \ \frac{ln\,N\,(\varepsilon)}{ln\,{}^{1}/_{\varepsilon}} \qquad (1)$$

**Measurement:** The photographic samples of each case of mimic and model were collected for total twenty cases of butterfly species (Table III). Total ten high- resolution photographs of both mimic and model for each case were selected from the authentic database and the blog of lepidopteron species. The each photograph was further converted into black and white resolution. The fractal dimension was calculated for each sample image, using MatLab (R2012b).

Only the fractal structure of counter part of the black portion in left wing pattern was considered to study the geometrical arrangement of the wing, formed by wing edge and the network of the wing veins, as because the fractal dimension includes information from all possible scales, from the tiniest details to the whole object. Therefore, in order to test how distinctive the fractal dimension may be, and to propose it as a way for differentiating groups of butterflies, the same measuring procedure was applied to butterflies from different, unrelated species. Thus the species which are closely resembles to each other at first site (Mimic and Model) can be distinguish by calculating the value of fractal dimension. To forward the present work the hypothesis may test and statistical data were obtained for the significant difference in fractal dimension value of butterfly species.

**Results:** The mean FD values from ten samples of each mimic and model species were calculated (Table-I). The data were analyzed statistical by applying student t-test and one way ANOVA at 0.05%. The established hypothesis at 95% confidence level was accepted for all the nineteen cases. The case 13 shows the T tabulated (4.458) > t (1.8124) critical, there is no possible distinguish in Mean FD value of Mimic (Common Wanderer) and Model (Yellow glassy Tiger).

Table I. Mean FD value

| Mean FD values | | |
|---|---|---|
| **Case No** | **Mimic** | **Model** |
| 1 | 1.81883 | 1.71885 |
| 2 | 1.72712 | 1.76756 |
| 3 | 1.7696 | 1.7324 |
| 4 | 1.7657 | 1.80067 |
| 5 | 1.694 | 1.71607 |
| 6 | 1.7985 | 1.8273 |
| 7 | 1.82211 | 1.83171 |
| 8 | 1.87179 | 1.82429 |
| 9 | 1.78799 | 1.81883 |
| | 1.86767 | |
| | 1.82615 | |
| 10 | 1.86219 | 1.86825 |
| 11 | 1.84346 | 1.81652 |
| | | 1.88298 |
| 12 | 1.87027 | 1.7746 |
| 13 | 1.79448 | 1.82728 |
| 14 | 1.8082 | 1.80638 |
| 15 | 1.78551 | 1.87713 |
| | 1.79448 | |
| 16 | 1.80541 | 1.84925 |
| | 1.79918 | |
| 17 | 1.79918 | 1.80098 |
| | 1.79417 | |
| 18 | 1.81222 | 1.8038 |
| | 1.83549 | |
| 19 | 1.83549 | 1.81069 |
| 20 | 1.74129 | 1.70134 |

The data were further drawn for the post-hoc statistical analysis with Cohn's test and also quantified with the values of effects size r (Fig. 1). The r values at different level shows the effective distinguished in mean FD values between mimic and model. The statistical data suggests the mean difference is not by chance and there is significant difference between the mean FD value of mimics and models.

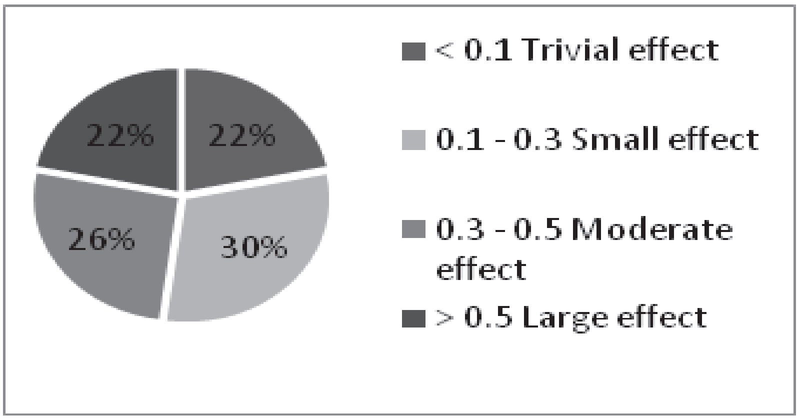


Fig.1 Total species with percent effect size r

**Discussion:** Total twenty mimicry case studies were put forward for the measurement of fractality and fractal dimension of their left hand wing pattern and it has been found that there is significant difference between mean FD value of mimic and model, this can be considered as an important fact when the measurement is based on geometrical pattern from all possible scale of an object and from tiniest point to large scale at different magnifications. Thus fractal dimension may applied as an effective tool to distinguished the different species which are look similar at the first sight (Mimic and Model).

With respect to the family Nymphalidae and Papilionidae, comprises more species diversity in different geographical region, they show the similar appearance in the form of mimicry (Batesian or Mullerian). The palatable species (Mimic) follow the subtle appearance of unpalatable and intense color species (Model) to get protection from the predators.

The range of FD value of mimic species (Minimum 1.6940 to Maximum 1.8717, Variation 0.00186) and for model (Mininimum 1.70134 and Maximum 1.88298, Variation 0.00264) shows that the species (Mimics and Models) which are from different families may share some common features. The data of FD value reflects the geometrical complexity in wing structure, were all the species are found to have some common developmental raw materials. This particular constrains can be contracted from the study of Nympalid ground plane [11]. The statistical analysis supports the findings and can be drawn as the basic geometry of butterfly wing may modify during the course of evolution with natural selection, it would also have an effect of selective agents like predation [5]. The present findings also supports the on-off mechanisms of genes, which also one of the important factor for species diversity [4].

The findings related to mimic cases found in subspecies of Mimic crescent (Table III) butterflies and the sex-linked mimicry found in Papilio Polytes (Table I) butterflies are agree with the other data suggesting that the characteristics that defines the species are often the product of natural selection [14].

It may will be that the tool of fractal dimension of the wing's pattern can be used to identify were the developmental raw material is common features and further only modified by natural selection.

This is the preliminary investigation, about the study of biological structure, in terms of their geometry by introducing the mathematical model. Although these results shows the fractality and fractal dimension of complex biological structure, they say nothing about the mechanism involve in this kind of particular pattern. Withstanding this critic, the study was established to initiate the application of mathematical programmer, for identifying group where, the developmental raw material is common feature. Our findings are interesting, and also raise some questions about the topology that leads behind the biological structure. Thus the study warrants further work, need to established the hypothesis to broaden the basis, regarding the genetically aspects that relate with evolutionary pattern. Table II shows sample photographs of some of the mimic cases.

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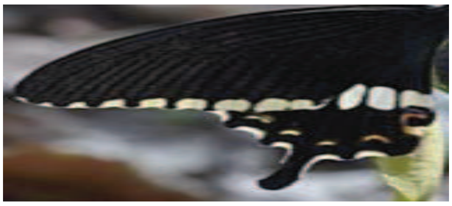
Table II Sample photographs of some mimic species

**Mimic**

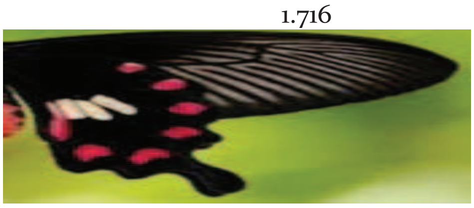


Papilio Polytes
(Female Cyrus)
1.694

**Model**

Papilio Polytes
(Male)
1.716

Papilio Polytes
(Female Stichius)
1.7657



Pachliopta aristolochiae
1.80067

**Mimic 1**

Elymnias malelas
1.8222

**Model**

Euploea muleiber
1.8038

**Mimic 2**

Chilasa peradoxa
1.83549

**Table III. Twenty cases of Mimicry with Mimic and Model [2], [6], [8].**

| | Mimic | | | | Model | | | |
|---|---|---|---|---|---|---|---|---|
| Case No | Common name | Scientific name | Family | Subfamily | Common name | Scientific name | Family | Subfamily |
| 1 | Mimic Crescent | Eresia pelonia Ithomia | N | Nymphalinae | Lenea Clearwing | Callithomia alexirrhoe thornax. | N | Danainae |
| 2 | Mimic Crescent | Eresia pelonia callonia | N | Nymphalinae | Mimic Crescent | Hypothyris mansuetus meterus | N | Nymphalinae |
| 3 | Red spotted Purple | Limenitis arthemis | N | Limnitidae | Pipevine Swallowtail | Battus philenor | P | Papilioninae |
| 4 | Common Mormon | Papilio Polytes Stichius (Female) | P | Papilioninae | Common Rose | Pachliopta aristolochiae | P | Papilioninae |
| 5 | Common Mormon | Papilio Polytes Cyrus (Female) | P | Papilioninae | Common Mormon | Papilio polytes (Male) | P | Papilioninae |
| 6 | Tailed Redbrest | Papilio bootes (Female) | P | Papilioninae | Rose Windmill | Byasa latreillei | P | Papilioninae |
| 7 | Jezebel Palmfly | Elymnias vasudeva | N | Satyrinae | Red spot Jezebel | Delias descombesi | Pi | Pierinae |
| 8 | Danaid Eggfly | Hypolimnas misippus (Female) | N | Nymphalinae | Plain Tiger | Danaus chrysippus | N | Danainae |
| 9 | Common Palmfly | Elymnias hypermnestra (Female) | N | Satyrinae | Common Tiger | Danaus plexippus | N | Danainae |
| | Tamil Lacewing | Cethosia nietneri mahratta | N | Heliconiinae | | | | |
| | Leopard Lacewing | Cethosia cyana | N | Heliconiinae | | | | |
| 10 | Lesser Mime | Papilio epycides | P | Papilioninae | Glassy Tiger | Parantica aglea | N | Danainae |
| 11 | Tawny Mime | Papilio agester | P | Papilioninae | Chestnut Tiger | Parantica sita | N | Danainae |
| 12 | Spotted Zebra | Paranticopsis magarus | P | Papilioninae | Dark Blue Tiger | Tirumala limniace | N | Danainae |
| 13 | Common Wanderer | Parenonia valeria (Female) | Pi | Pierinae | Yellow glassy Tiger | Parantica aspasia | N | Danainae |
| 14 | Great Zebra | Graphium xenocles | P | Papilioninae | Chocolate Tiger | Danais melaneus | N | Danainae |
| 15 | Dark Wanderer | Pareronia ceylanica | Pi | Pierinae | Blue Tiger | Danais limniace | N | Danainae |
| | Common Wanderer | Parenonia valeria | Pi | Pierinae | | | | |
| 16 | Common Mime | Papilio clytia | P | Papilioninae | Common Indian Crow | Euploea core | N | Danainae |
| | Malbar Raven | Papilio dravidarum | P | Papilioninae | | | | |
| 17 | Common Raven | Papilio Castor | P | Papilioninae | Double branded Crow | Eploea sylvester | N | Danainae |
| | Great Eggfly | Hypolimnas bolina | N | Nymphalinae | | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 18 | Spotted Palmfly | Elymnias malelas | N | Satyrinae | Striped blue Crow | Euploea mulciber | N | Danainae |
| | Great blue Mime | Papilio paradoxa | P | Papilioninae | | | | |
| 19 | Great blue Mime | Papilio paradoxa | P | Papilioninae | Magpie Crow | Eupleoa radamanthus | N | Danainae |
| 20 | Monarch Butterfly | Danaus plexippus | N | Danainae | Viceroy Butterfly | Limenitis archippus | N | Limenitidinae |

N = Nymphalidae, P = Papilionidae, Pi= Pieridae

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