WEAKLY NONLINEAR CONVECTION IN A VARIABLE VISCOSITY FLUID SATURATED POROUS MEDIUM UNDER INTERNAL HEATING AND TEMPERATURE MODULATION EFFECTS

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Abstract: The effects of temperature dependent viscosity, internal heat source, temperature modulation and thermo-mechanical anisotropy on heat transport in a low-porosity medium are studied using the Ginzburg-Landau model. The amplitudes of temperature modulation at both the boundaries are considered to be very small, and the disturbances are expanded in terms of power series of amplitude of convection. A weak nonlinear stability analysis has been performed for the stationary mode of convection, and heat transport in terms of the Nusselt number, which is governed by the nonautonomous Ginzburg-Landau equation, is calculated. The effects of thermorheological parameter, internal Rayleigh number, amplitude and frequency of modulation, thermo mechanical anisotropies and Darcy-Prandtl number on heat transport have been analyzed and depicted graphically. It is found that increments in the values of thermo-rheological parameter and internal Rayleigh number result in enhancement of heat transport in the system. Further, temperature modulation can be used to control the heat transport effectively by a mechanism that is external to the system.

Keywords: Ginzburg-Landau model, Temperature modulation, Anisotropic porous media, Temperature-dependent viscosity, Internal heating.

Introduction: Venezian (1969) was first to perform a linear stability analysis of Rayleigh-Bénard convection, for the case of small amplitude temperature modulation. The analog of this problem in porous media was introduced by Caltagirone (1976). Chhuon and Caltagirone (1979), Malashetty and Wadi (1999), Bhadauria (2007a,c), Bhadauria (2008a,b), Bhadauria and Sherani (2008), Bhadauria and Suthar (2009), Bhadauria and Srivastava (2010) are some other works available in the literature considering temperature modulation.

The present paper deals with internal heating effect on thermal instability over a porous medium. It is an extension of our recent paper Bhadauria and Palle Kiran (2013), in which we have investigated the effect of temperature dependent viscosity on heat transport in anisotropic porous layer. The effect of temperature dependent viscosity extensively investigated by Nield (1996), Holzbecher (1998), Rees et al. (2002), Siddheshwar and Chan (2004), Vanishree and Siddheshwar (2010), Srivastava et al.(2013) for different physical configuration of the problem.

Internal heat generation is very important in many applications including storage of radioactive materials, combustion and fire studies, geophysics, reactor safety analysis. However, there are only few studies available in which the effect of internal heating on convective flow investigated: Bhattacharya and Jena, (1984), Rionero and Straughan, (1990), Rao and Wang (1991), Parthiban and Patil (1997), Herron Isom (2001), Joshi et al.(2006), Bhadauria et al.(2011), Bhadauria (2012) and Bhadauria et al. (2013a,b,c). Our interest here is the thermo-rheological parameter which arises due to temperature-dependent viscosity, and can be used to control the convective flow. Also internal heating of the system can play an important role in controlling the convection. Therefore, it is with this motive that we have investigated the effect of temperature modulation, thermo-rheological parameter, and internal heating on heat transport.

Governing Equations: we consider an infinitely extended horizontal anisotropic porous layer, saturated by variable viscosity newtonian fluid, confined between two impermeable boundaries at $z = 0$ and $z = d$, which are heated from below and cooled from above in a time periodic manner. we choose cartesian frame of reference with origin in the lower boundary and the $z=d$ axis vertically upward given in *fig.1.* it is assumed that the mechanical properties and thermal properties in x and y-directions are same. further, darcy law and the oberbeck-boussinesq approximation are taken to be applicable.

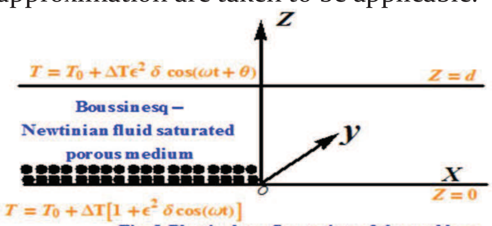

Fig .1.Physical configuration of the problem.

The equations which describe this system under above considerations are given by

$$\nabla \cdot \vec{q} = 0 \qquad (1)$$

$$\frac{\rho_0}{\phi}\frac{\partial \vec{q}}{\partial t} = -\nabla p + \rho \vec{g} - \mu(\mathrm{T}) K . \vec{q}, \qquad (2)$$

$$\gamma \frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla)\mathrm{T} = \nabla.(\kappa_T . \nabla T) + Q\left(T - T_0\right), \qquad (3)$$

$$\rho = \rho_0[1 - \beta_T (T - T_0)], \qquad (4)$$

$$\mu(T) = \frac{\mu_0}{1+\varepsilon^2\delta_0(T-T_0)} \quad (5)$$

where $\vec{q}$ is velocity (u, v, w), Q is internal heat source, ϕ is the porosity of the medium, $\mu(\mathrm{T})$ temperature dependant variable viscosity, $K = K_x^{-1}(\hat{i}\hat{i}+\hat{j}\hat{j}) + K_z^{-1}\hat{k}\hat{k}$ is the permeability tensor, $\kappa_T = \kappa_{Tx}(\hat{i}\hat{i}+\hat{j}\hat{j}) + \kappa_{Tz}\hat{k}\hat{k}$ is the thermal diffusivity, T is temperature, β_T is thermal expansion coefficient, γ is the ratio of heat capacities, ρ is the density, while ρ_0 is the reference density, g is the acceleration due to gravity.

III. MATHEMATICAL FORMULATION

The externally imposed surface temperature are considered as

$$\mathrm{T} = \mathrm{T}_0 + \Delta T\left[1+\varepsilon^2\delta\cos(\omega t)\right] \qquad \text{at } z=0$$

$$= \mathrm{T}_0 - \Delta T\varepsilon^2\delta\cos(\omega t+\theta) \quad \text{at } z=d \quad (6)$$

where δ is small amplitude of temperature modulation, ω is modulation frequency and is the phase difference. The thermo-rhelogical relationship *Eq.(5)* is guided by Nield (1996). The basic state is assumed to be quiescent and the quantities in the state are given by

$$\vec{q} = q_b(z),\ \rho=\rho_b(z,t),\ p=p_b(z,t),\ T=T_b(z,t), \quad (7)$$

$$\frac{\partial p_b}{\partial z} = g\rho_b, \quad (8)$$

$$\frac{\partial T_b}{\partial t} = \kappa_{T_z}\frac{\partial^2 T_b}{\partial z^2} + Q\,(T_b - T_0), \quad (9)$$

$$\rho_b = \rho_0[1-\beta_T(T_b-T_0)]. \quad (10)$$

The basic temperature field *Eq.(9)* has been solved subject to the thermal boundary conditions *Eq.(6),* and the solution is found to be of the form

$$T_b(\mathrm{z,t}) = T_s(z) + \varepsilon^2\delta\,\mathrm{Re}\{T_1(\mathrm{z,t})\}, \quad (11)$$

where $T_s(z)$ the steady temperature is field and T_1 is the oscillating part, while Re stands for the real part. We impose finite amplitude perturbations on the basic state in the form:

$$\vec{q} = q_b + \vec{q}',\ \rho=\rho_b+\rho',\ \mathrm{p}=\mathrm{p}_b+\mathrm{p}',\ T=T_b+T' \quad (12)$$

Substituting *Eq. (12)* into *Eqs. (1) - (5),* we get the following equations

$$\nabla\cdot\vec{q}' = 0, \quad (13)$$

$$\frac{\partial\overrightarrow{q'}}{\partial t} = -\frac{\nabla p'}{\rho_0} + \frac{\rho'}{\rho_0}\vec{g} - \nu\mu(\mathrm{T})K.\overrightarrow{q'}, \quad (14)$$

$$\frac{\partial T'}{\partial t} + (\vec{q}'\cdot\nabla)T' + \mathrm{w}'\frac{\partial T_b}{\partial z} = \nabla.(\kappa_T.\nabla)T' + QT', \quad (15)$$

$$\rho' = -\rho_0\beta_T T', \quad (16)$$

$$\mu(T) = \frac{1}{1+\varepsilon^2\delta_0(T_b-T_0)} \quad (17)$$

We consider only two-dimensional disturbances in our study, and hence the stream function ψ may be introduced in the form:

$$\mathrm{u}' = \frac{\partial\psi}{\partial z},\ \mathrm{w}' = -\frac{\partial\psi}{\partial x} \quad (18)$$

Eliminating the pressure term p from *Eq. (14)* and then non dimensionlizing the equations using the following scales:

$$(x',y',z') = d(x^*,\mathrm{y}^*,\mathrm{z}^*),\ \mathrm{t}=\frac{d^2}{\kappa_{T_z}}t^*\quad q' = \frac{\kappa_{T_z}}{d}q^*$$

$$T'=\Delta TT^*\ \psi = \kappa_{T_z}\psi^*,\ \text{and}\ \omega^* = \frac{\kappa_{T_z}}{d^2}\omega,$$

we get the non-dimensional governing equations in the form:

$$\frac{1}{\mathrm{Pr}_D}\frac{\partial\nabla^2\psi}{\partial t} = -\overline{\mu}(\mathrm{T})\nabla_\xi^2\psi - Ra_T\frac{\partial T}{\partial x} - \frac{1}{\xi}\frac{\partial\overline{\mu}}{\partial z}\frac{\partial\psi}{\partial z}, \quad (19)$$

$$-\frac{\partial\psi}{\partial x}\frac{\partial T_b}{\partial z} - (\nabla_\eta^2 + R_i)T = -\frac{\partial T}{\partial t} + \frac{\partial(\psi,T)}{\partial(\mathrm{x,z})}, \quad (20)$$

where $\overline{\mu}(T) = \frac{1}{1+\varepsilon^2VT}$, $Ra_T = \frac{\beta_T g\Delta TdK_z}{\nu\kappa_{Tz}}$ is thermal Rayleigh number, $\mathrm{Pr}_D = \frac{\phi\nu d^2}{K_z\kappa_{Tz}}$ is Darcy Prandtl number, $R_i = \frac{Qd^2}{\kappa_{Tz}}$ is internal Rayleigh number, $V = \delta_0\Delta T$ is the thermo-rheological parameter or variable viscosity parameter. The non-dimensional basic temperature $T_b(z,t)$ which appears in the *Eq. (20)* can be obtained from *Eq.(11)* as

$$\frac{\partial\mathrm{T}_b}{\partial z} = f_1(z) + \varepsilon^2\delta[f_2(\mathrm{z,t})], \quad (21)$$

where

$$f_1 = -\frac{\sqrt{R_i}\cos\sqrt{R_i}(1-\mathrm{z})}{\sin\sqrt{R_i}}, \quad (22)$$

$$f_2 = \mathrm{Re}[fe^{-\omega t}],\ f = [\mathrm{A}(m)e^{mz} + \mathrm{A}(-m)e^{-mz}],$$

$$m = \sqrt{\lambda^2 - R_i},\ \lambda^2 = -i\omega,\ A(m) = m\frac{e^{m\theta}-e^{-m}}{e^m - e^{-m}}.$$

To keep the time variation slow we have rescaled the time t by using the time scale $\tau = \varepsilon^2$ t . Also the value of γ has been taken equal to one for simplicity. It is to be noted that the system is not being considered to be over stable as we are interested only in stationary convection. We re-write the nonlinear *Eqs. (19) - (20)* in the matrix form as given below:

$$\begin{bmatrix} -\overline{\mu}(T)\nabla_\xi^2 & Ra_T\frac{\partial}{\partial x} \\ -f_1\frac{\partial}{\partial x} & -(\nabla_\eta^2+\mathrm{R}_i) \end{bmatrix}\begin{bmatrix}\psi \\ T\end{bmatrix} = \begin{bmatrix} -\frac{\varepsilon^2}{\mathrm{Pr}_D}\frac{\partial\nabla^2\psi}{\partial\tau} - \frac{1}{\xi}\frac{\partial\overline{\mu}}{\partial z}\frac{\partial\psi}{\partial z} \\ -\varepsilon^2\frac{\partial T}{\partial\tau} + \frac{\partial(\psi,T)}{\partial(\mathrm{x,z})} + \varepsilon^2\delta f_2\frac{\partial\psi}{\partial x} \end{bmatrix}$$

(23)

The boundary condition to solve *Eq. (23)* are:

$$\psi = 0, \quad T = 0 \quad \text{on} \quad z=1$$
$$\psi = 0, \quad T = 1 \quad \text{on} \quad z=0 \tag{24}$$

Heat Transport:

We introduce the following asymptotic expansion in Eq.(23):

$$Ra_T = Ra_{0c} + \varepsilon^2 R_2 + .. \tag{25}$$
$$\psi = \varepsilon\psi_1 + \varepsilon^2\psi_2 + \varepsilon^3\psi_2 + .. \tag{26}$$
$$T = \varepsilon T_1 + \varepsilon^2 T_2 + \varepsilon^3 T_3 + .. \tag{27}$$

where R_{0c} is the critical Rayleigh number at which the onset of convection takes place in the absence of modulation. Using *Eqs.(25)-(27)* in *Eq. (23)* we solve the system for different orders of ε. **At the lowest order**, we have

$$\begin{bmatrix} \nabla_\xi^2 & Ra_{0c}\dfrac{\partial}{\partial x} \\ -f_1\dfrac{\partial}{\partial x} & -(\nabla_\eta^2 + \mathrm{R}_i) \end{bmatrix}\begin{bmatrix}\psi_1 \\ T_1\end{bmatrix} = \begin{bmatrix}0 \\ 0\end{bmatrix}, \tag{28}$$

The solution of the lowest order system subject to the boundary conditions *Eq.(24)*, is

$$\psi_1 = A(\tau)\sin(\mathrm{k}_c \mathrm{x})\sin(\pi\,\mathrm{z}), \tag{29}$$

$$T_1 = \frac{4\pi^2 k_c}{\delta_R^2\left(R_i - 4\pi^2\right)}A(\tau)\cos(k_c x)\sin(\pi\,\mathrm{z}), \tag{30}$$

where, $\delta_R^2 = \eta k_c^2 + \pi^2 - R_i$. The critical value of the Rayleigh number for the onset of stationary convection is calculated numerically, and the expression is given by:

$$Ra_{0c} = \frac{\delta_R^2\delta_\xi^2\left(4\pi^2 - R_i\right)}{4\pi^2 k_c^2}, \tag{31}$$

$$k_c = \left[\frac{\pi^2\left(\pi^2 - R_i\right)}{\eta\xi}\right]^{\frac{1}{4}}, \tag{32}$$

where $\delta_\xi^2 = \mathrm{k}_c^2 + \dfrac{\pi^2}{\xi}$. For the system without internal heating we have:

$$Ra_{0c} = \frac{\delta_R^2\delta_\xi^2}{k_c^2} \tag{33}$$

$$k_c = \frac{\pi}{[\eta\xi]^{\frac{1}{4}}}, \tag{34}$$

which are the classical results of Epherre, (1975). If we take $\xi = \eta = 1$, then we get the classical results of Lapwood (1948), for isotropic porous medium.

At the second order, we have

$$\begin{bmatrix} \nabla_\xi^2 & Ra_{0c}\dfrac{\partial}{\partial x} \\ -f_1\dfrac{\partial}{\partial x} & -(\nabla_\eta^2 + \mathrm{R}_i) \end{bmatrix}\begin{bmatrix}\psi_2 \\ T_2\end{bmatrix} = \begin{bmatrix}R_{21} \\ R_{22}\end{bmatrix} \tag{35}$$

where

$$R_{21} = 0, \tag{36}$$

$$R_{22} = \frac{\partial\psi_1}{\partial x}\frac{\partial T_1}{\partial z} - \frac{\partial\psi_1}{\partial z}\frac{\partial T_1}{\partial x}. \tag{37}$$

The second order solutions subjected to the boundary conditions *Eq.(24)*, is obtained as

$$\psi_2 = 0 \tag{38}$$

$$T_2 = -\frac{2\pi^3 \mathrm{k}_c^2}{\delta_R^2\left(4\pi^2 - R_i\right)^2}A^2(\tau)\sin(2\pi\,\mathrm{z}), \tag{39}$$

The horizontally averaged Nusselt number, *Nu,* for the stationary convection (the mode considered in this problem) is given by:

$$Nu(\tau) = 1 + \frac{\left[\dfrac{\mathrm{k}_c}{2\pi}\displaystyle\int_0^{\frac{2\pi}{\mathrm{k}_c}}\frac{\partial T_2}{\partial z}dx\right]_{z=0}}{\left[\dfrac{\mathrm{k}_c}{2\pi}\displaystyle\int_0^{\frac{2\pi}{\mathrm{k}_c}}\frac{\partial T_b}{\partial z}dx\right]_{z=0}} \tag{40}$$

Substituting *Eqs. (21)* and *(39)* in *Eq.(40)* and simplifying, we get

$$Nu(\tau) = 1 + \frac{4\pi^2 k_c^2\sin\sqrt{R_i}[\mathrm{A}(\tau)]^2}{\delta_R^2\left(4\pi^2 - R_i\right)^2\sqrt{R_i}\cos\sqrt{R_i}}. \tag{41}$$

We must note here that f_2 is effective at $O(\varepsilon^2)$ and affects *Nu(t)* through $\mathrm{A}(\tau)$ as shown next.

At the third order, we have

$$\begin{bmatrix} \nabla_\xi^2 & Ra_{0c}\dfrac{\partial}{\partial x} \\ -f_1\dfrac{\partial}{\partial x} & -(\nabla_\eta^2 + \mathrm{R}_i) \end{bmatrix}\begin{bmatrix}\psi_3 \\ T_3\end{bmatrix} = \begin{bmatrix}R_{31} \\ R_{32}\end{bmatrix} \tag{42}$$

where

$$R_{31} = -\frac{1}{\mathrm{Pr}_D}\frac{\partial}{\partial\tau}\nabla^2\psi_1 - Vf_1\nabla_\xi^2\psi_1 - \frac{Vf_1}{\xi}\frac{\partial\psi_1}{\partial z} - \left(R_2 + 2Ra_{0c}Vf_1\right)\frac{\partial T_1}{\partial x}, \tag{43}$$

$$R_{32} = -\frac{\partial T_1}{\partial\tau} + \frac{\partial T_2}{\partial z}\frac{\partial\psi_1}{\partial x} + \delta f_2\frac{\partial\psi_1}{\partial x}, \tag{44}$$

Substituting ψ_1, T_1 and T_2 into *Esq.(42)-(43),* and applying the solvability condition for the existence of third order solution, we get the Ginzburg-Landau equation in the form

$$Q_1\frac{\partial A(\tau)}{\partial\tau} - Q_2\,\mathrm{A}(\tau) + Q_3\,\mathrm{A}(\tau)^3 = 0. \tag{45}$$

$$Q_1 = \frac{\delta^2}{\mathrm{Pr}_D\,\delta_\xi^2} + \frac{1}{\delta_R^2},\quad Q_3 = \frac{\pi^2 k_c^2}{2\delta_R^2\left(4\pi^2 - R_i\right)},$$

$$Q_2 = \frac{R_2}{Ra_{0c}} - \frac{4\pi^2 V\left(\cos\sqrt{R_i} - 1\right)}{\sqrt{R_i}\sin\sqrt{R_i}\left(4\pi^2 - R_i\right)} + \frac{2\pi^2 V\sqrt{R_i}\left(\cos\sqrt{R_i} - 1\right)}{\xi\nabla_\xi^2\sin\sqrt{R_i}\left(4\pi^2 - R_i\right)} - \frac{\left(4\pi^2 - R_i\right)}{2\pi^2}\delta I_1$$

and $\quad I_1(\tau) = \displaystyle\int_{z=0}^{1} f_2\sin^2(\pi z)dz.$

The above Ginzburg-Landau equation *(45)* has been solved numerically using the inbuilt function NDSolve of Mathmatica 8.0, subject to the suitable initial condition *A(0) = a0*, where a0 is the chosen initial amplitude of convection. In our calculations we may assume $R_2 = Ra_{0c}$, to keep the parameters to the minimum.

Results And Discussion: A weakly nonlinear stability analysis has been performed to investigate the combined effect of internal heating and temperature modulation on thermal instability in a temperature dependent viscous fluid saturated closely packed anisotropic porous medium. The effect of temperature modulation on the Bénard-Darcy system has been assumed to be of order $O(\varepsilon^2)$. This means, we consider only small amplitude temperature modulation. The work of Nield (1996), has been used for the thermo-rheological relationship of temperature dependant viscosity of the fluid.

The temperature modulation has been considered in the following three cases:

- In-phase modulation (*IPM)* ($\theta = 0$).
- Out-phase modulation (*OPM*) ($\theta = \pi$).
- Only Lower boundary modulated (*LBMO*) ($\theta = -i\infty$).which means only lower boundary temperature is modulated, the upper boundary is kept at constant temperature.

Since the porous medium is assumed to be closely packed, the Darcy-model is considered in governing equation. The parameters that arise in this study of convection and influence the heat transport are *Ri, V,* Pr_D, ξ, η, δ, ω and θ. The first five are related to the properties of fluid and porous media, and last three are external mechanism for controlling convection.

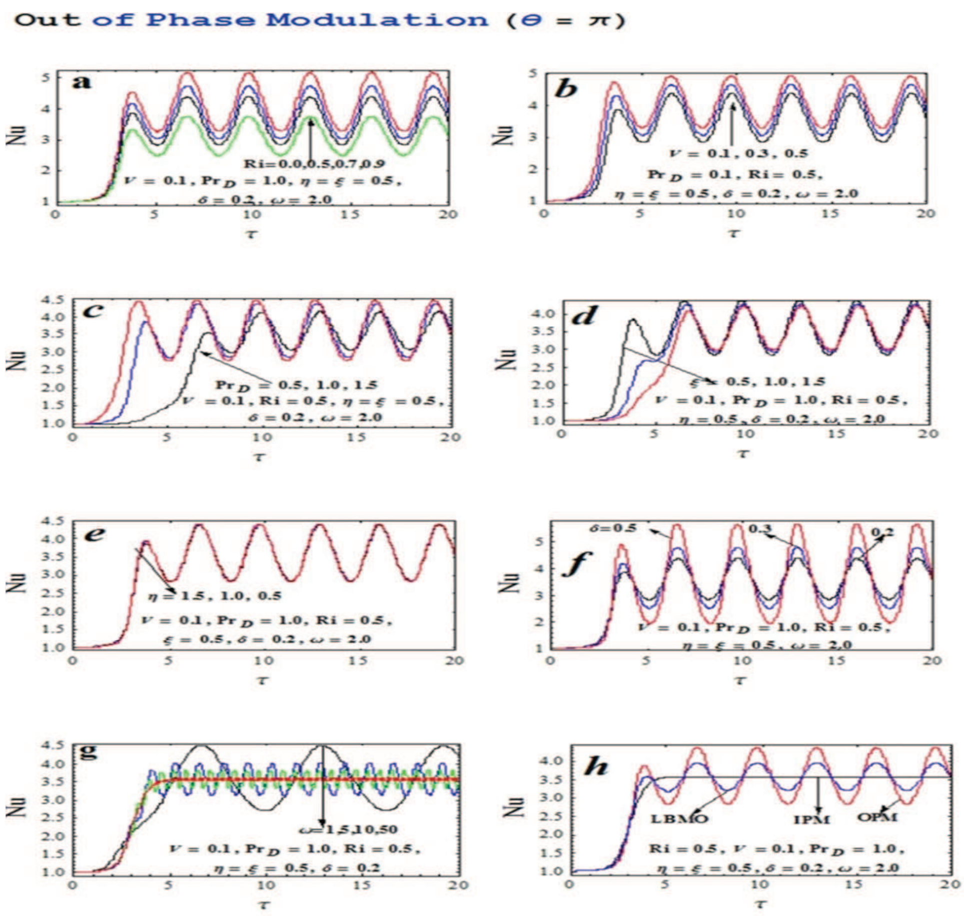

Fig .3 : Plots of Nu verses τ for out of phase modulation.

Vadasz (1998), pointed out that in unmodulated case there are many real situations in which the value of Pr_D is very large, therefore one can neglect the time-derivative term in Darcy *Eq.(19)*. Further, he points out that there are however some modern porous medium applications, such as mushy layer in solidification of binary alloys and fractured porous medium, where the value of Pr_D may be considered of the order unity, therefore the time-derivative term in the present study has been retained. This is the reason that we have kept the values of Pr_D around one in our calculations, and retained the local acceleration term $\frac{1}{Pr_D}\frac{\partial q}{\partial \tau}$. The values of *Ri* are considered to be moderate so that it will not affect the effect of temperature modulation of the system by dominating it otherwise. The values of δ are consider very small around 0.2, since we are studying the effect of small amplitude modulation on the heat transport. Also, the effect of low frequencies, is maximum, on the onset of convection as well as on the heat transport, therefore the modulation of temperature is assumed to be

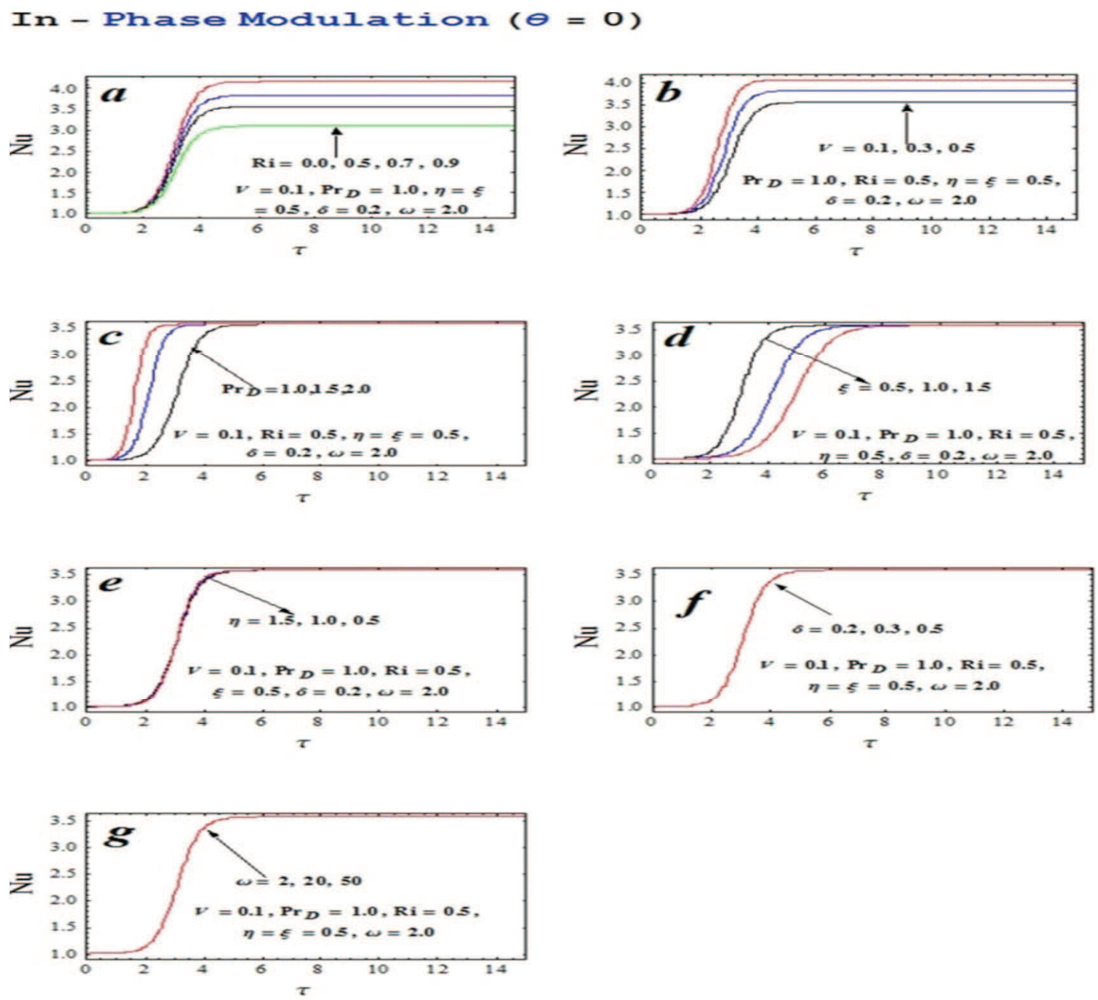

Fig .2 : Plots of Nu verses τ for in phase modulation.

of low frequency. Further, the value of thermo-rheological parameter, V is also considered to be small.

Figures (2), (3) and (4) present the numerical results for $Nu(\tau)$, obtained from the expression in *Eq.(41)* by solving the amplitude *Eq.(45)*. It is clear to see the expression in *Eq.(45)* in conjunction with *Eq.(41)* that $Nu(\tau)$ is a function of internal heating parameter Ri, Darcy-Prandtl number Pr_D, thermo-rheological parameter V, thermo-mechanical anisotropy parameters ξ and η, and the amplitude and frequency of modulation, δ and ω. The effect of each type of modulation on heat transport is shown in *Figs.(2-4)* wherein the plots of Nusselt number $Nu(\tau)$ verses τ are presented. It is found from the figures that the value of $Nu(\tau)$ starts with one and remains constant for quite some time, thus showing the conduction state initially. Then the value of $Nu(\tau)$ increases with time, thus showing the convection state and finally becomes constant on further increasing τ thus achieving the study state.

Figs.(2a-g) present the results for *IPM*. We observe that $Nu(\tau)$ increases with individual and collective increases in the internal Rayleigh number Ri, Darcy-Prandtl number Pr_D and thermo-rheological parameter V, but decreases with increase in mechanical anisotropy ξ. Thus, there is appreciable enhancement in heat transport on increasing Ri, Pr_D and V thereby advancing the onset of convection. However, the heat transport decreases on increasing ξ, thus delaying the convection. The effects of Pr_D and ξ on heat transport diminish at large values of time τ. Further, the amplitude of modulation δ and the frequency modulation ω both have negligible effects on heat transport in this case. Further, an increment in thermal anisotropic parameter η, decreases $Nu(\tau)$ initially and then increases with time. Thus the effect of mechanical and thermal anisotropy is found to be opposite at large time, compatible with the results of Epherre (1975), Kuznetsov and Nield (2008) and Bhadauria and Kiran (2013) obtained for the unmodulated case. However at small time τ, the effect of thermal anisotropy η is similar to ξ which is just opposite to the unmodulated case. This is because of interplay between internal heat generation and temperature modulation of the boundaries. It is also found that the results obtained for the modulated (*IPM)* and unmodulated cases are qualitatively same. Further, we observe that the effect of internal heating on heat transport in the convective system *Fig. (2a),* the magnitude of *Nu(τ)* is greater than that in the absence of internal heating. The physical reason for this is that internal heating advances the onset of convection.

In *Figs.(3a-g),* we have depicted the variation of *Nu(τ)* with time τ for out of phase modulations. It is found that *Nu(τ)* starts with one, increases with increasing time *Nu(τ)* and then becomes oscillatory. However, on further increasing the time, it approaches the steady state. We observe from *Figs.(3a-e)* that the effects of *Ri, Va, V,* ξ and η on heat transport are found to be similar to those of *IPM*. Further, we found in *Fig.(3f)* that the effect of amplitude of modulation is to increase the magnitude of *Nu(τ),* thus increasing the heat transport and advancing the convection.

Also, from *Fig.(3g),* we observe that an increase in the frequency of modulation decreases the magnitude of *Nu(τ),* and so the effect of frequency of modulation on heat transport diminishes. At high frequency the effect of temperature modulation on thermal instability disappears altogether. This result agrees quite well with the linear theory results (Venezian, 1969), where the correction in the critical value of Rayleigh number due to temperature modulation becomes almost zero at high frequencies. *LBMO* results followed by *OPM*, due to this we have not presented figures in case of *LBMO*, but the following results can be observed in figure *(3h).*

$$Nu_{IPM} < Nu_{LBMO} < Nu_{OPM}$$

Most of the results are qualitatively similar to the results obtained by Bhadauria et al. (2012,2013c) and Siddheshwar et al. (2012a,b,2013).

Conclusions: The following conclusions are made:

- Effect of *IPM* is negligible on heat transport in the system.
- In the case of *IPM*, the effect of δ and ω are also found to be negligible on heat transport.
- Effect of *Ri, V* and Pr_D is to enhance the heat transport for all three types of modulations.
- Effect of mechanical anisotropy ξ is to decrease the heat transport for all three types of modulations.
- Effect of η on heat transport is negligible for all three types of modulations.
- Heat transport is more in the present case than in non-internal heating case of Bhadauria et al. (2012), Siddheshwar et al. (2013).
- In the case of *IPM*, *Nu* increase steadily for intermediate value of time τ and ultimately becomes constant when τ is large.
- In the case of *OPM* and *LBMO*, *Nu* shows an oscillatory nature.
- The thermo-rheological model of Nield (1996), gives physically acceptable results, namely, the destabilizing effect on Bénard-Darcy convection and thereby an enhanced heat transport.

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