g-PRE REGULAR AND g-PRE NORMAL TOPOLOGICAL SPACES

VIDYOTTAMA KUMARI, DR. THAKUR C. K. RAMAN

Abstract: In general topology, the notion of pre-open set, introduced by A.S. Mashour et al. [1982], has a significant role and the most important generalizations of regularity & normality appear as the notions of pre-regularity along with strong regularity [1983] and pre-normality as well as strong normality [1984] respectively.
In 1970, N. Levine projected the concept of so called g-closed sets in topological spaces in an independent way and studied its basic properties.
Since then many modifications of g-closed sets were defined and investigated by a large number of topologists. In 1996, Maki et al. introduced the concepts of gp-closed sets. The purpose of this paper is to study the classes of regular spaces & normal spaces, namely gp-regular spaces &gp-normal spaces which are a generalization of the classes of p-regular & p-normal spaces respectively. The paper also contains the behaviour of pre * —$T_{1/2}$ spaces whenever it is strongly regular or strongly normal. It also highlights the pre-topological property of a gp-normal pre- R_0 spaces.
Also, through this paper, a tribute is being paid to the renowned mathematician Professor M.E. Abd. El – Monsef who left for his heavenly abode on 13th August, 2014.

Introduction & preliminaries: Various new topological concepts& their basic properties have been defied & investigated using the notion of pre-open sets & pre-open, pre-continuous mappings(i.e. pre homeomorphism) as introduced by A.S. Mashhour et al. [1]. In 1998, T. Noiri et al. [2] studied generalized pre closed functions using generalized preclosed sets.

A subset A of a space (X, T) is known as a generalized pre-closed iff every open superset of A contains its pre-closure[2].

He concepts of g-pre –regular $& g-pre –normal spaces as, here, studied using generalized- pre- closed sets.

Definition (1.1)[2]: Asubset A of a space (X,T) is said to be generalized preclosed (briefly gp-closed) iffpcl(A) ⊆ U whenever A ⊆ U and U is open in X.

Definition (1.4)[6]: A space(X,T) is called pre * —$T_{1/2}$ if every gp-closed set in X is preclosed.

Definition (1.5): Afunction f:(X,T)→(Y,σ) is called pre continuous [1] (resp. pre irresolute[7]) if the inverse image of each open(resp. pre open) set of Y is pre open in X.

Definition (1.6)[8]: A bijective function f : (X,T)→(Y,σ) is called pre- homeomophism if f is M-preopen and pre-irresolute.

Definition (1.7) [5]: A space (X,T) is called strongly regular if for each preclosed set A & each point x ∉ A , there exist pre-open sets U & V such that x∈ U & A ⊂ V.

Definition (1.8): A space (X,T) is called strongly normal if for each pair of disjoint preclosed sets A & B, there exist pre-open sets U & V such that A ⊆U & B ⊆ V.Any other notion and symbol, not defined in this paper, may be found in the appropriate reference.

§2. g- pre regular spaces: This section introduces g-pre regular spaces in topological spaces.

Definition (2.1): A topological space (X,T) is said to be g-pre-regular (in short gp-regular) space iff every gp-closed set F and every point x ∉ F , there exist disjoint pre-open sets U & V such that F ⊂ U & x∈ V .Obviously, every gp-regular space is strongly regular space, but not conversely.

Lemma (2.2): A strongly regular pre * —$T_{1/2}$ space is gp-regular.

Proof: Let (X,T) be a strongly regular space as well as pre * —$T_{1/2}$ space. Since,(X,T) is a pre * —$T_{1/2}$ space, hence every gp-closed set in X is preclosed i.e. the class of gp-closed sets & pre-closed sets coincide. Now, (X, T) is strongly regular space which provides that for each preclosed set A & each point x ∉A , there exist disjoint pre-open sets U & V such that x∈ U & A ⊂ V. Combining these facts, it is concluded that for each gp-closed set A and each point there exist disjoint pre-open sets U & V such that A ⊂ U & x∈ V , which turns (X,T) to be a gp-regular.

Characterization criteria:

Theorem (2.3): A topological space (X,T) is gp-regular iff every gp-closed set F and every point x ∉

F,there exists pre-open sets U & V such that x∈ U, F ⊂ V and pcl(U) ∩ pcl(V) = ϕ.

Proof: Suppose thatF is a gp- closed set of a space (X, T) and x ∉ F. Since, (X,T) is a gp-regular space hence, there exist disjoint pre-open sets U & V such that F ⊂ V & x∈ U & U∩ V = ϕ. Obviously, U∩ V = ϕ⇒ U∩ pcl(V)= ϕ& pcl(U)∩ V = ϕ⇒ pcl(U)∩ pcl(V) = ϕ. Converse is not natural, so omitted.

Theorem (2.4): For a space (X, T) the following are equivalent:

(i) (X, T) is gp-regular.

(ii) for every x∈ X and for every gp- open set W containing x there exists a pre open set V such that pcl V⊆ W.

(iii) for every gp-closed set F and every point x ∉ F , there exists pre-open sets V such that pcl(V) ∩ F = ϕ.

Proof: The proof is as natural as exhibited in the case of the characterization of a normal space.

Hereditary property: The following lemmas are helpful in analyzing the hereditary property of gp-regular spaces:

Lemma (2.5): If X_o∈ αO(X,T) and A ∈ PO(X,T), then X_o∩ A ∈ PO(X_o , T_{X_0}),[5]

Lemma (2.6): Suppose B ⊆ A ⊆ X and (X,T) is a space. If A is open & gp-closed in (X,T) and B is a gp-closed in (A, T_A), then B is also gp-closed in (X,T).

Theorem (2.7): If (X,T) is a gp-regular space & Y is an open and gp-closed subset of (X,T) , then the subspace (Y, T_Y) is a gp-regular space.

i.e. gp-regularity is a hereditary property with respect to an open &gp-closed subspace.

Proof: The lemmas (2.5)&(2.6) are the base & hereditary criteria of a regular space is the motivation for the establishment of the theorem.

Preservation theorem: the gp-regularity of a space is preserved under a bijective, gp irresolute and M-pre –open mapping as established in the following theorem.

Theorem (2.8): If f : (X,T)→(Y,σ) be a bijective, gp-irresolute and M – pre-open mapping from a gp-regular (X,T), then (Y,σ) is also gp-regular.

§3. g -pre normal spaces: The weak form of normality called gp-normality in topological spaces is being introduced and studied in this section.

Definition (3.1): A topological space (X,T) is said to be g-pre-normal (in short gp-normal) space iff for any pair of disjoint gp-closed sets A & B, there exist disjoint pre-open sets U & V such that A ⊆ U & B ⊆ V.

Transformation of gp-normal space into a gp- regular space occurs only when it is a pre-R_o space as described through the following theorem(3.2).

Theorm (3.2): Every gp-normal , pre- R_o space is gp-regular

Characterization criteria: The following theorems are innunciated to charecterize a gp-normal space.

Theorm (3.3): A topological space (X,T) is gp-normal iffeverypair of disjoint gp-closed sets A and B there exist a pair of pre-open sets U & V such that A ⊆ U & B ⊆ V. and pcl(U) ∩ pcl(V) = ϕ.

Theorem(3.4) For a space (X,T) the following are equivalent:

(i) (X,T) is gp-normal..

(ii) for every gp- closed set F and every open set G containing F, there exists a pre open set V such that F⊆ U ⊆ pcl (U)⊆ G.

Proof: the proof is based upon the required definition & procedure according to the text.

Hereditary criteria: gp-normality is hereditary property with respect to an open and gp-closed subspace.

Theorem (3.5): If (X,T) is a gp-normal space and Y is an open &gp-closed subset of (X,T), then (Y, T_Y) is a gp-normal subspace.

Preservation criteria: the gp-normality of a space is preserved under a bijective, gp-irresolute amd M—pre-open mapping as expressed in following theorem.

Theorem (3.6): If f : (X,T)→(Y,σ) be a bijective, gp-irresolute and M – pre-open mapping from a gp-normal (X,T), then (Y,σ) is also gp-normal..

Conclusion: Transformation of a strongly regulars pace into a gp-regular space under the criteria of being pre* —$T_{1/2}$ has been discussed.

Transformation of a gp-normal space into a gp-regular space under the criteria of beingpre-R_o has also been analyzed.

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VidyottamaKumari/Assistant Professor/ Deptt. Of Mathematics/
R.V.S College Of Engineering & Technology/Jamshedpur/Jharkhand/India/Research Scholar/
Kolhan University/Chaibasa/ Jharkhand/ India/k.vidyottama@gmail.com.
Dr. Thakur C. K. Raman/ Associate Professor & Head/Deptt. Of Mathematics/ Jamshedpur Workers' College/
Jamshedpur/Jharkhand/India/thakurckraman@gmail.com.