APPLICATION OF MATHEMATICS IN ECONOMIC ANALYSIS USING LINEAR PROGRAMMING

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Abstract: Economics is the study of how resources are used as well as an analysis of the decisions made in allocating resources and distributing goods and services. Mathematics is the language of numbers and symbols that can be used to logically solve problems and precisely describe size, quantity and other concepts. Some complex problems could not be described and complicated problems could not be acted upon without the language of mathematics and its support of logical processes to solve problems. Mathematical modeling is used in economic analysis to study existing economic relationships and it helps economists study what-if scenarios to see what might happen to the economy if a certain action is applied. Economic concepts and relationships can be measured in mathematical indexes, formulas and graphs. Several areas of mathematics can be utilized in economic analysis, including linear algebra, calculus, and geometry. Since economic concepts can be complex, it is important to use care in representing data and relationships in isolation. Results can be misinterpreted based on the representation of data. Thus in this paper an attempt is made to analyze the role of mathematics and linear programming in Economics

Introduction: Traditionally, application of mathematics had been restricted to the physical sciences, and the theories in the social sciences had been neglected, but in these days we notice that mathematical economics is flying high. We also observe that the articles on mathematical economics and fewer points on economic theory, occupy more prominent place in the economics journals.

Arguments given in the favor of mathematics look attractive in the first instance but they are not free of problems. It is argued by many economists that mathematical models are recognized in providing a rational approach to solving many of the problems in decision making, allocation, and forecasting1.
Mathematical models present theoretical work in their own language, which is a tool of communication, but we know that a language must be simple and easily understood to be appreciated, but as a language, mathematics is not simple and easy.

Mathematization of Economics: The major development of the second quarter of 20th century in the field of economics was the mathematization of economics. An economist of 19th century cannot even understand the economic journals of present times. Starting from the microeconomics theory, macroeconomics, international trade, economic development, public finance and all the other branches of economics have been changed into a number of equations.

The development of mathematical economics is not revolutionary step. It took several centuries to develop the present stage of mathematical economics. Sir William Petty (1623-1687) is believed to be the first participant in this field. He used the terms of symbols in his studies, but he was not successful. The first successful attempt was made by an Italian, named Giovanni Ceva (1647—1734). After these earlier developments, Antoine Augustin Cournot (1801-1877) made use of symbols in his theory of wealth.

Alfred Marshall in his *"Principles of Economics"* (1890), and Irving Fisher in his Ph.D. thesis *"Mathematical Investigations in The Theory of Value and Prices"* showed a great interest in mathematical formulation of the economic theory5. After their work, such a race began in this field that everyone specialized in mathematics and with less knowledge in the economic theory jumped in this new field and more and more articles started publishing with excessive use of mathematics and lacking theory.

Difference between Mathematical and Literary Economics: It is almost as hard to define mathematics as it is to define economics. An easy definition of economics is given by Jacob Viner, "Economics is what economists do"6, so we can say that mathematics is what mathematicians do. Mathematical economics is not an individual branch of economics in the sense that international trade, public finance, or urban economics, but it is an approach to economic theory.

In mathematical economics, mathematical symbols and equation are used in the statement of the problem. Since mathematical economics is just an approach to economic analysis, it must not differ from the non-mathematical approach in the conclusion but we observe entirely opposite situation and here the problems starts. The major difference between mathematical economics and literary economics is that in the former, the assumptions and conclusions are described in mathematical symbols and equations whereas, in the later, words and sentences are used to achieve the desired goal.

Significance of Economics Models: The term "Model" is very common in economics. It can be defined as a set of assumptions from which the conclusions can be drawn. In simple words we can say that model is simply a representation of some aspects of the real world. Economic theory is descriptive as well as analytical.

It does not give us complete descriptions of economic phenomenon, but by making certain assumptions, we can construct models. The models then help in representing reality and help in understanding the characteristics of economic behavior. In economic models, we can use both mathematical and theoretical approaches. The choice between these approaches depends upon the personal preference of the research person. If the model is math mathematical, it will consist of a set of symbols and equations, designed to describe the structure of the model.

Linear Programming: Linear programming is a recently devised technique for providing specific numerical solutions of problems which earlier could be solved only in vague qualitative terms by using the apparatus of the general theory of the firm.

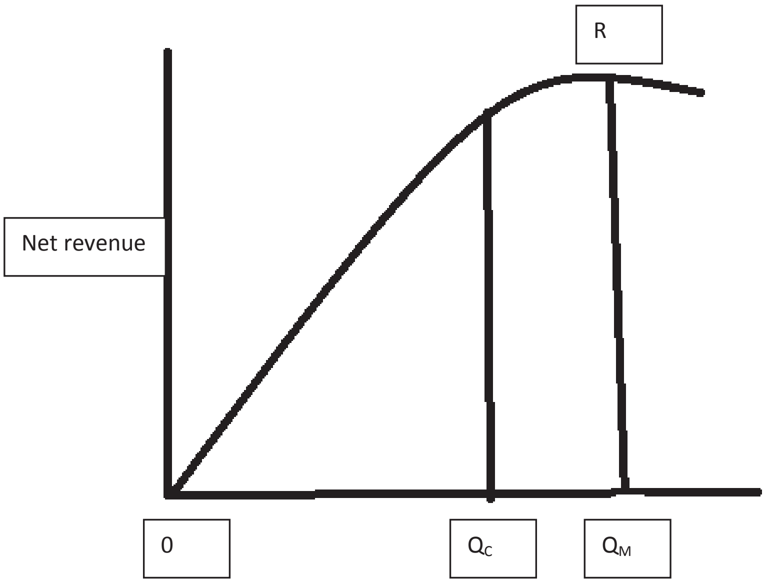
Linear programming has thus helped to bridge the gap between abstract economic theory and managerial decision-making in practice. Programming is a technique which is applied for finding maximum or minimum values in problems confronting the decision-making authorities, subject to certain constraints or side-conditions which limit decisions. In fact, such problems which require determination of maxima or minima could also be solved with the help of Calculus. But it is found that in the presence of certain side-conditions which are not exact but limit the requirements, the application of Calculus breaks down. Programming, therefore, helps to find out maximum or minimum values when side-conditions are inequalities and not equations.

In case the expression to be maximized or minimized and the inequalities involving side-conditions are 1^{st} degree, the method of solving the problem is known as linear programming.
With given cost-function and price we can locate the point of maximum profit by determining a point at which marginal profitability is zero. For profit maximization the conditions are:

$$\frac{\partial \pi}{\partial Q} = 0 \ (1st - order\ condition) and$$

$$\frac{\partial^2 \pi}{\partial Q^2} < 0 \ (2nd\ - order\ condition)$$

Supposing that such point occurs at R in the above figure, at which output must be OQ_M. But this level of output may not be attained by the firm in question, due to certain constraints at different stages of production. Suppose that the firm cannot increase its output beyond OQ_C. In the presence of the constraints, OQ_M level of output in the context of maximizing the total profit becomes irrelevant because the firm cannot attain it even though it might like to do so. As such our problem is to find out whether the point of maximum attainable profit is really OQ_C and is represented by some other point to its left. Also all the points to the left of R will not satisfy the maximization conditions, and the application of Order conditions of Calculus break down.



Problems of such nature as are concerned with the efficient use of limited resources which serve as side-conditions as to achieve the desired objective can be solved with the help of Programming Technique.

The term 'Linear' implies that the relationship involved in the problems must be linear, i.e., of 1st degree while the term "Programming" refers to determination of particular programme which gives the line of action which would achieve the desired objective

Assumptions: The application of L.P. (Linear Programming) rests on several basic assumption and requirement.

A Well Defined Objective Function: The decision maker wants to achieve certain goal; this may be maximization of net income, profit, sales or minimization of cost. The objective function should be expressed as a linear function of decision variables. In the example given above, the objective is to maximize net revenue by choosing appropriate values of the decision variables –output of the products I and II.

Presence of Constrains in the Activities: The L.P. technique can be applied to the problem which involves certain kinds of constraints in their activities. Such constraints usually are because of technological limitations, or capacity and resource limitations. They exist primarily because resources are limited.

Non – Negativity of Decision Variables: Along with these structural constraints we have "Non- negativity" constrains which assumes negative value of the variables is not possible in the solution of L.P. problems.

Linear Relations in The Problem: L.P. problem are always started in linear relation. In fact, functional relationship among variables may not be actually linear in many cases, but problems cannot be solved with help of this technique.

Constant Prices: In order that we deal with linear relations only, prices of inputs and outputs are to be assumed given and constant during a given time period. Hence prices are not considered as variables throughout the programming exercise.

Divisibility: Activity levels are permitted to take fractional or interger values.

L.P. techniques enables us to deal with problems involving objectives functions which are either required to be maximumised or minimized under certain given constraints. We now consider a problem in which objective function is to be minimized under given conditions.

Let us assume a hypothetical problem in which a person requires a certain minimum amount of each of 3 types of vitamins per day in order to maintain his health. We also assume that these 3 types of vitamins are found in specific quantities in the 2 food items. If the price per unit of each food item is known, our problem is to find the combination of food items which must be purchased so that the daily vitamin requirements are met and simultaneously the cost of the food items purchased is minimum with the following given data.

Vitamins (Types)	Food items containing Vitamins per kg		Minimum daily
	I	II	
A1	10	4	20
A2	5	5	20
A3	2	6	12
Price per Kg. of food item	Rs.0.60	Rs. 1.00	-

The table indicates one unit of Food I contains 10 units of vitamin A1, 5 units of vitamin A2 and 2 units of vitamin A3. Similarly food II contains 4 units of A1, 5 units of A2 and 6 units of A3. The daily minimum requirement for a person to maintain his health is 20 units of A1, 20 units of A2 and 12 units of A3.

With the prices of both the food items given, and assuming that x1, quantity of Food I and x2 quantity of food II will satisfy the minimum requirement of the vitamins, our problem can be put in algebraic form:

Minimize: $f= 0.6x_1+x_2$

$10x_1+4x_2>20$

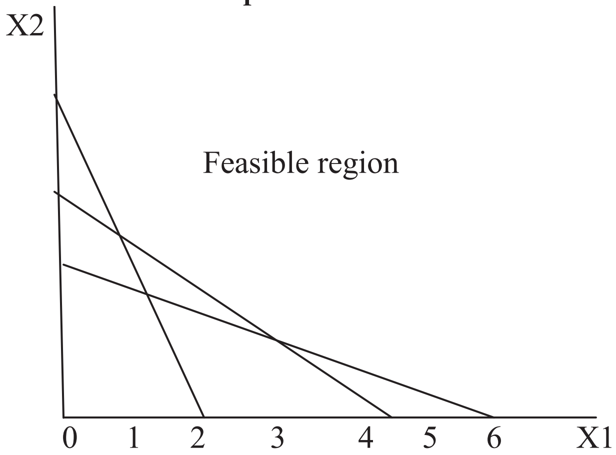
$5x_1+5x_2>20$

$2x_1+6x_2>12$, and

$x_1, x_2 >0$

Each structural constraint is of >type. This is because to maintain his health the man in question must consume minimum of 20 units of A1 and not less than that amount. Same is applicable to the case of vitamins A2 and A3.

Graphical Method:



We plot all the 3 constraint inequalities as explained under the maximization problem. Here again because of non-negativity restrictions, we need only to consider the positive quadrant as specified.

Since each constraint is of > type, solution to our problem would lie above these lines of constraints. For this reason now the shaded area above the lines will be our feasible region and each point in this region will contain feasible solution.

Again to minimize cost we must select the lowest possible iso-cost line while still staying in the feasible region. The above diagram gives the boundaries of feasible region. In order that objective function should be minimum, the movement of iso-cost should be as indicated by the arrows.

CC1 is the only iso-cost line satisfying out both conditions:
(i) It gives the solution which is feasible (since R is the extreme point of feasible region); and
(ii) It involves least cost (since any iso-cost line below it would violate all the constraints).
How to find point R? Since this point is the interaction of constraint lines of vitamin A2 and vitamin in A3, we solve the equations:

$$5X_1+5X_2= 20 \qquad X_1=3$$
$$2X_1+6X_2=12 \qquad X_2=1$$

Hence solution to our problem is that a person should purchase 3 units of food 1 and one unit of food II. These amounts of food will not only provide the required intake of vitamins but will also involve minimum cost to the consumer.

References:

1. Mathematics for Economists- Mehta & Madnani
2. Allen R.G,D, **Mathematical Analysis for Economists,** London Macmillan and Co, Ltd, (1962).
3. Bell, John Fred **A History of Economic Thought,** N.Y Ronald Press Company, (1967).
4. Blaine, Roberts and Schultz, David L., **Modern Mathematics and Economic Analysis,** N.Y, W.W.Norton and Co, Inc., (1973).

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