IMPROVED FUZZY LINEAR SPACE

R. K. PARMAR

Abstract : Fuzzy mathematics is a branch of mathematics based on fuzzy theory. Fuzzy theory was put forward by L. A. Zadeh. In mathematics fuzzy sets have triggered new research topics in connection with category theory, topology, algebra, analysis. Fuzzy sets are also part of a recent trend in the study of generalized measures and integrals, and are combined with statistical methods. Fuzzy set-based techniques are also an important ingredient in the development of information technologi**es**. Fuzzy rule-based modeling has been combined with other techniques such as neural nets and evolutionary computing and applied to systems and control engineering, with applications to robotics, complex process control and supervision. Sudarshan Nanda defined fuzzyfield and fuzzy linear space first time. R. Biswas pointed out the mistakes and redefined fuzzy linear space. In this paper we improve the fuzzy linear space and prove some basic theorems of linear space.

Keywords : Fuzzy set, Linear Space, membership function, intersection

Introduction : The concept of fuzzy sets was first introduced by Zadeh in 1965, and then the fuzzy sets have been used in the reconsideration of classical mathematics. Nanda introduced the concepts of fuzzy field and fuzzy linear space and gave some results. Biswas pointed out that Proposition (iii) given by Nanda was incorrect and redefined fuzzy field and fuzzy linear space. In this paper, we first introduce the concepts of fuzzy linear space given by Nanda and Biswas. Next, we generalize the concepts of fuzzy linear spaces and prove theorems.

1.Fuzzy Linear Space: Let F be a fuzzy field in a field (X, +, .) with membership function F(λ). Let Y be a linear space over F and V be a fuzzy subset of Y with membership function V(x). Then, V is called a fuzzy linear space in Y if following postulates are satisfied:

(i) $V(x+y) \geq \min\{V(x), V(y)\}, \quad \forall\, x, y \in Y$

(ii) $V(\lambda x) \geq \min\{F(\lambda), V(x)\}, \forall\, \lambda \in F$ and $\forall\, x \in Y$

(iii) $V(0) = 1$

In case F is an ordinary field then, F (λ) = 1 and hence

$V(\lambda x) \geq \min\{1, V(x)\}, \forall \lambda \in F$ and $x \in Y$

$= V(x)$

Hence, for F to be an ordinary field, the (ii) postulate may be considered as

$V(\lambda x) \geq V(x), \forall \lambda \in F$ and $x \in Y$

Now we will analyze the definition of fuzzy linear space introduced by Nanda.

Let us consider the case when F and V both are classical set. Then , we have F (λ) = 1, V(x) = 1 and V(y) = 1 for all x , y ∈ F and λ ∈ F.

Hence, from condition (i), we have

$V(x + y) = 1 \Rightarrow x + y \in V$

Thus, we get that x, y ∈ V ⇒ x + y ∈ V.

Further, from condition (ii), we get

$V(\lambda x) \geq \min\{1,1\} = 1$

i.e. $V(\lambda x) = 1 \Rightarrow \lambda x \in V$. That is, $x \in V, \lambda \in F \Rightarrow \lambda x \in V$.

It follows that V is closed under addition and scalar multiplication.

Thus, on the basis of above discussion we arrive at the conclusion that the definition of fuzzy linear space has been considered in such a way that when F and V both are considered as an ordinary subset, V turns out to be a subspace of Y.

The concept of fuzzy linear space was redefined by R. Biswas in 1989 as he was not satisfied by the definition of fuzzy linear space proposed by Nanda. Biswas redefined the notion of fuzzy linear space, as follows:

Fuzzy Linear Space Redefined: Let F be a fuzzy field in a field (X, +, .) with membership function F(λ). Let Y be a linear space over F and V is a fuzzy subset of Y with membership function V(x). Then, V is said to be a fuzzy linear space in Y if the following conditions are satisfied:

$V(x + y) \geq \min\{V(x), V(y), \forall\, x, y \in Y$

$V(\lambda x) \geq \min\{F(\lambda), V(x)\}, \forall \lambda \in F, x \in Y$

Biswas deleted (iii) condition given by Nnada.

Improved Definition Of Fuzzy Linear Space: Let F be a fuzzy field in a field (X, +, .). Let Y be a linear space over F and V is a fuzzy subset of Y. Then V be called as a fuzzy linear space in Y if the following condition are satisfied:

$V(x + y) \geq V(x). V(y), \forall\, x, y \in Y$

$V(\lambda x) \geq F(\lambda .V(x), \forall \lambda \in F$ and $x \in Y$

If F is an ordinary field (in particular if F = X), then F (λ) = 1.Then the, (ii) condition reduces to

$V(\lambda x) \geq V(x), \forall \lambda \in Y$

Now we claim that our definition of fuzzy linear space is more general than that of Biswas.

For, if V is a fuzzy linear space in Y in the sense of Biswas, then

$V(x + y) \geq \min\{V(x), V(y)\} \geq V(x).V(y), \forall\, x, y \in Y$

and $V(\lambda x) \geq \min\{F(\lambda), V(x)\} \geq F(\lambda).F(x), \forall \lambda \in F, x \in Y$

It follows that V is a fuzzy linear space according to our definition when it is a fuzzy linear space according to Biswas.

Next, we suppose that V is a fuzzy linear space in Y according to our definition. That is $V(x + y) \geq V(x).V(y)$

And $V(\lambda x) \geq F(\lambda).V(x)$

But $V(x).V(y) \geq \min\{V(x),V(y)\}$

and $F(\lambda).V(x) \geq \min\{F(\lambda),V(x)\}$ are not true.

Therefore, V is not a fuzzy linear space in view of Biswas, while it is a fuzzy linear space under our definition.

Theorem 1. The intersection of two fuzzy linear spaces is again a fuzzy linear space.

Proof: Let U and V are two fuzzy linear spaces in a linear space Y over a fuzzy field F. Then, for all x, y $\in$Y and $\lambda \in F = X$, we have

$U(x + y) \geq U(x).U(y)$

and $U(\lambda x) \geq U(x)$

also $V(x + y) \geq V(x).V(y)$

$V(\lambda x) \geq V(x)$

Now, we have

$(U\cap V)(x + y) = \min\{U(x + y),V(x + y)\}$

$\geq \min\{U(x).U(y),V(x).V(y)\}$

and $(U\cap V)(x).(U\cap V)(y) = \min\{U(x),V(x)\}.\min\{U(y),V(y)\}$

$= \min\{U(x).U(y),U(x).V(y), V(x).U(y),V(x).V(y)\}$

Since $\min\{U(x).U(y),V(x).V(y)\} \geq \min\{U(x).U(y),U(x).V(y), V(x).U(y),V(x).V(y)\}$

$\therefore (U\cap V)(x + y) \geq (U\cap V)(x).(U\cap V)(y)$

Also $(U\cap V)(\lambda x) = \min\{U(\lambda x). V(\lambda x)\}$

$\geq \min\{U(x), V(x)\}$

$= (U\cap V)(x)$

$(U\cap V)(\lambda x) \geq (U\cap V)(x)$

This establishes the claim that intersection of two fuzzy linear spaces U and V is again a fuzzy linear space over a field F = X.

Theorem 2. Intersection of any family of fuzzy linear spaces is a fuzzy linear space.

Proof: Let $\{V_i : i \in I\}$ be a family of fuzzy linear spaces in a linear space Y over a fuzzy filed F in a filed X.

Let $\bigcap_{i\in I} V_i = V$.

Then, we claim that V is fuzzy linear space in Y.

We have for all x, y$\in$ Y

$V(x + y) = \bigcap_{i\in I} V_i (x + y)$

$= \inf_{i\in I} V_i(x + y)$

$\geq \inf_{i\in I} [V_i(x).V_i(y)]$, since $\forall\ i \in I$, V_i is a FLS

$= \inf_{i\in I} V_i(x). \inf_{i\in I} V_i(y)$

$= \bigcap_{i\in I} V_i(x) . \bigcap_{i\in I} V_i(y)$

$= V(x).V(y)$

i.e. $V(x + y) \geq V(x) . V(y) ,\forall\ x , y \in Y$

Also $V(\lambda x) = \bigcap_{i\in I} V_i(\lambda x),\forall\ \lambda \in F$

$= \inf_{i\in I} V_i(\lambda x)$

$\geq \inf_{i\in I} [F(\lambda).V_i(x)]$

$= F(\lambda) . \inf_{i\in I} V_i(x)$.

$= F(\lambda) . \bigcap_{i\in I} V_i(x)$

$= F(\lambda).V(x)$

$\therefore V(\lambda x) \geq F(\lambda).V(x),\forall\lambda\in F$ and $x \in Y$

Therefore, V i.e. $\bigcap_{i\in I} V_i$ is a fuzzy linear space over Y.

References:

1. Biswas,R [1989] "Fuzzy Field and Linear Spaces Redefined", Fuzzy Sets and Systems,33,pp.257-259
2. Nanda, Sudarshan [1986] "Fuzzy Fields and Fuzzy Linear Spaces" Fuzzy Sets and Systems,9,pp 89-84
3. S. Vijaybalaji [2015] "A cubic set theoretical approach to linear space" Hindawi
4. C.Umadevi, K.R.Mohan, A.K.Sharma, A Comparative Study Of The Discharge Characteristics Of (Pvp/Pva) Polymer Blend Electrolyte Films Doped With Different Potassium Salts.; Engineering Sciences international Research Journal : ISSN 2320-4338 Volume 3 Issue 2 (2015), Pg 38-44
5. Publishing Corporation, volume 2015,article id 523129.
6. Rosenfeld, A [1971] "Fuzzy Groups',J.Math.Anal.Appl.35 pp 512-517.
7. Singh, M.K.[2006] " A Note on Fuzzy Algebraic Structures" ,J. ofUltra Scientist of Physical Sciences,V.18,No. 03,pp 483-488

R. K. Parmar /Asstt. Professor , Department of Mathematics,
Bhagat Singh Govt. P. G. College, Jaora (M.P.), India.